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DECELERATION OF A STRONG SHOCK WAVE BY A TRANSVERSE MAGNETIC FIELD AT SUBSTANTIAL MAGNETIC REYNOLDS NUMBERS

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UDC 538.6

Strong ionizing shock waves can be used for obtaining strong magnetic fields and high-powered short-duration pulses of electrical energy. The motionless gas ahead of the wave front is cold and nonconducting, while behind the front the gas moves at high speed and has considerable electrical conductivity because of its high temperature. The interaction of the high-velocity conductive stream produced by the ionizing shock wave with the magnetic and electric field can be utilized in various applications, one of which is magnetic cumulation, i.e., transformation of the energy of the wave into the energy of a compressed magnetic field and its subsequent utilization for various purposes [1]. Another subject of great interest is the utilization of ionizing shock waves moving in a transverse magnetic field for studying the effects of the T layer [2].

A theoretical investigation of the interaction of an ionizing shock wave with a transverse magnetic field at substantial magnetic Reynolds numbers can be carried out in the most complete form by using direct finite-difference methods which presuppose the utilization of implicit conservative calculation schemes [3, 4].

Analytic solutions of such problems are only partial solutions and serve to make clear only the qualitative aspects of the processes taking place.

One of the modifications of the numerical methods for solving the problem of the interaction of an ionizing shock wave with a magnetic field is based on singling out a hyperbolic subsystem from the original equations and solving this subsystem by the method of characteristics in combination with a direct numerical solution of the other equations. Although such an approach imposes additional restrictions on the calculation model for the problem (no viscosity, etc.), it makes it possible to use an explicit difference scheme for solving a nonlinear hyperbolic subsystem, which makes the time required for solving the problem on an electronic computer considerably shorter than the computation time for direct difference algorithms.

1. Calculation Model and Transformation of the Original System of Equations. We consider a model (Fig. 1) similar to the current grid used in the experiments of [5]. There is a plane region bounded by a highly conductive Π-shaped frame, into which a strong ionizing shock wave enters with velocity $\mathbf{w}(w, 0, 0)$. Within the frame, for $x \geq 0$, we have a magnetic field $\mathbf{B}_e(0, B_e, 0)$. The conductive gas behind the wave moves with velocity $\mathbf{u}(u, 0, 0)$. Because of the Faraday effect, in this gas there are currents with density $\mathbf{j}(0, 0, j)$, closing along the frame, which strengthen the magnetic field $\mathbf{B}(0, B, 0)$ within the frame and decelerate the gas and the shock wave. The energy of the shock wave is transformed into magnetic field energy and Joule losses in the gas.

The model described above also corresponds to a coaxial channel with a relatively small gap along the radius (the z direction in Fig. 1).

In the analysis given below we shall neglect the Hall effect, since the pressure behind the wave front is high; we shall also neglect the electrical boundary effects, since the electrical connection is short-circuited. These assumptions enable us to use a one-dimensional approximation in solving the problem.

The interaction of the gas behind the wave front with the magnetic field is described by nonstationary equations of motion, continuity, and energy, Maxwell's equations, and Ohm's law, which in dimensionless form, for an ideal perfect gas, are

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 22-28, March-April, 1977. Original article submitted October 9, 1975.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{\mathrm{Eu}}{\rho}\frac{\partial p}{\partial x} = -Sj\frac{B}{\rho}; \tag{1.1}$$

$$\frac{\partial \rho}{\partial t} + \rho\frac{\partial u}{\partial x} + u\frac{\partial \rho}{\partial x} = 0; \tag{1.2}$$

$$\frac{\partial p}{\partial t} + \gamma p\frac{\partial u}{\partial x} + u\frac{\partial p}{\partial x} = \frac{S}{\mathrm{Eu}}(\gamma - 1)\frac{j^2}{\sigma}; \tag{1.3}$$

$$\frac{\partial B}{\partial x} = \mathrm{Re}_m j; \tag{1.4}$$

$$\frac{\partial E}{\partial x} = \frac{\partial B}{\partial t}; \tag{1.5}$$

$$j = \sigma(uB + E), \tag{1.6}$$

where p, ρ, σ, γ are the pressure, density, conductivity, and adiabatic exponent of the gas, respectively; E is the electric field strength; $\mathrm{Eu} = p_0/\rho_0 u_0^2$ is the Euler number; $S = \sigma_0 B_0^2 l/\rho_0 u_0$ is the interaction parameter; $\mathrm{Re}_m = \mu_0 \sigma_0 u_0 l$ is the magnetic Reynolds number; the subscript 0 indicates the basis values of the parameters; time is referred to l/u_0 and dimensions, to l. The equations are written in the laboratory system of coordinates. Changing to a noninertial system bound to the wave front would complicate the problem in this case. The conductivity of the gas is an arbitrary function of p and ρ:

$$\sigma = \sigma(p, \rho). \tag{1.7}$$

In principle, the method admits of specifying σ in the form $\sigma(p, \rho, j, B)$ with the nonequilibrium effects and the Hall effect taken into account. The use of a perfect-gas model is also nonessential for the analysis. To investigate nonperfect gases and gas compositions, we must use a general energy equation and a concrete equation of state of the gas.

Equations (1.1)-(1.7) must be supplemented by relations between the parameters on the front of the shock wave. We assume that ionization of the gas takes place immediately behind the front and that the wave is gas-dynamic. In addition, the wave must be sufficiently strong to ionize the gas. Therefore, we can use the known relations on the front in the form

$$\rho_w/\rho_{00} \cong (\gamma + 1)/(\gamma - 1); \tag{1.8}$$

$$\frac{p_w}{p_{00}} \cong \frac{2\gamma}{\gamma + 1}\mathrm{M}_w^2; \tag{1.9}$$

$$w/u_w \cong (\gamma + 1)/2. \tag{1.10}$$

Here the subscript w corresponds to the parameters immediately behind the front, the subscript 00 refers to the parameters of the unperturbed gas, and $M_w = w/a_{00}$, where a_{00} is the speed of sound in the gas ahead of the wave front.

Formulas (1.8)-(1.10) are necessary for finding the trajectory of the wave front and determining the boundary conditions on the front line.

The fundamental equations (1.1)-(1.6) form a quasilinear parabolically degenerate system [6]. A solution must be constructed in a region with a variable boundary, whose position is determined by integrating w with respect to t.

The authors of [7] propose to solve a system similar to (1.1)-(1.6) by artificially replacing the hyperbolic form and then using the method of characteristics. This substitution is carried out by introducing parametric

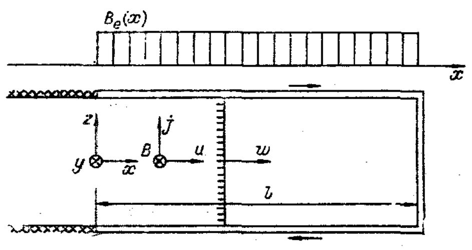

Fig. 1

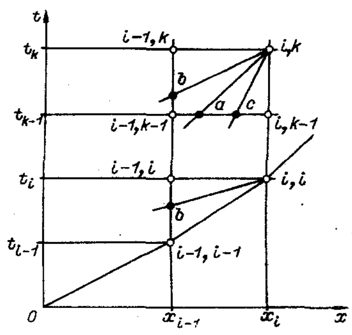

Fig. 2

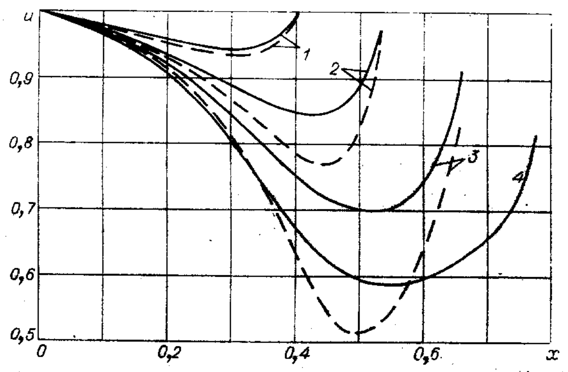

Fig. 3

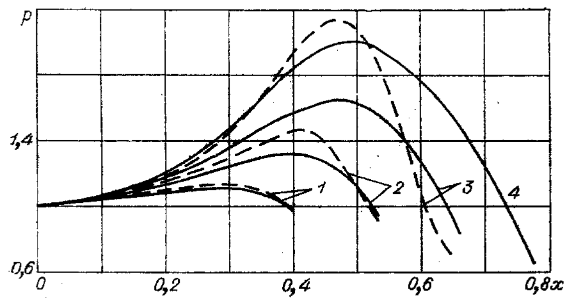

Fig. 4

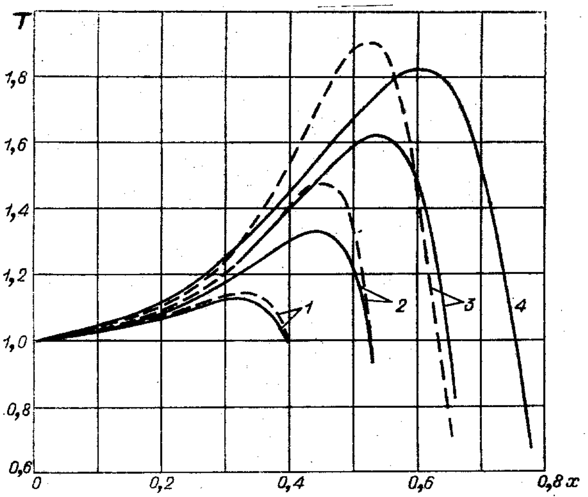

Fig. 5

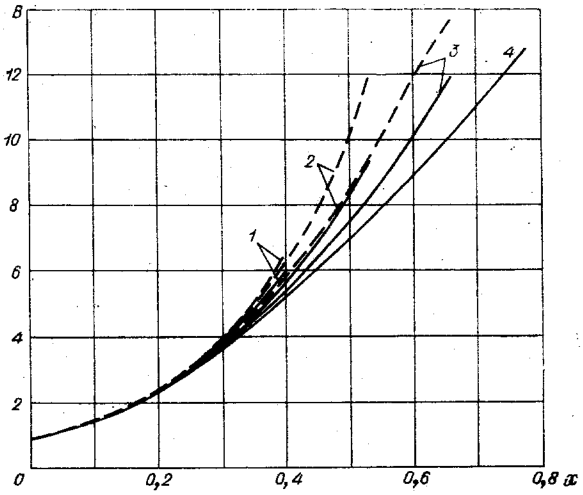

Fig. 6

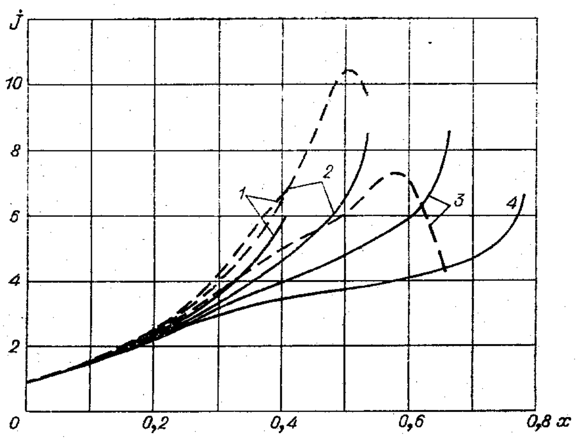

Fig. 7

functions representing some average values of the spatial derivatives of the magnetic induction. Similar transformations may be found effective for a number of problems; however, in the present case they require massive iterations for real calculations.

Another possibility of using the method of characteristics for solving the original system is based on the fact that the first four equations form a hyperbolic sybsystem whose solution can be combined block by block with the solutions of Eqs. (1.5) and (1.6).

It should be noted that in problems with a potential electric field, when instead of Eqs. (1.5), (1.6) we use the equation of the external electric circuit, we can also use an intermediate solution of a hyperbolic subsystem and then join it to the equation of the external circuit [8].

In [9] it was pointed out that the method of characteristics can be used for constructing computation algorithms for the system (1.1)-(1.6) by introducing some conditional parametric function analogous to inductance.

We first investigate the hyperbolic subsystem (1.1)-(1.4) describing the variation of u, p, ρ, and B.

The characteristic normal form equivalent to this is

$$dB/dx = \mathrm{Re}_m j \quad \text{along} \quad (dx/dt)_{\mathrm{I}} = \infty; \tag{1.11}$$

$$\mathrm{Eu}\,dp/dt - A^2 d\rho/dt = [(\gamma - 1)/\sigma] S j^2 \quad \text{along} \quad (dx/dt)_{\mathrm{II}} = u; \tag{1.12}$$

$$\mathrm{Eu}\,dp/dt + A\rho\,du/dt = Sj[(\gamma - 1)(j/\sigma) - AB] \quad \text{along} \quad (dx/dt)_{\mathrm{III}} = u + A; \tag{1.13}$$

$$\mathrm{Eu}\,dp/dt - A\rho\,du/dt = Sj[(\gamma - 1)(j/\sigma) + AB] \quad \text{along} \quad (dx/dt)_{\mathrm{IV}} = u - A, \tag{1.14}$$

where $A = \sqrt{\mathrm{Eu}\gamma p/\rho}$ is the dimensionless speed of sound.

The second subsystem (1.5), (1.6), if we take account of the condition $E(0, t) = 0$, reduces to the single equation

$$j = \sigma\left(uB + \frac{\partial}{\partial t}\int_0^x B\,dx\right). \tag{1.15}$$

In formulating the boundary conditions, we assume that at the inlet the gas parameters remain unperturbed and that the magnetic induction is equal to the induction of the external field B_e. The first assumption is made because the gas moves at supersonic speed ahead of the front, while the second is determined by the selected geometry of the model, in which all currents are closed to the right of the inlet and do not affect B(0, t). The inlet values of the parameters are considered the basis quantities in Eqs. (1.1)-(1.6). Thus, we have

$$u(0, t) = p(0, t) = \rho(0, t) = B(0, t) = 1, \qquad E(0, t) = 0. \tag{1.16}$$

The condition for E agrees with the fact that $E \equiv 0$ when $x < 0$. The conditions for u, p, and ρ are not strict conditions, since at the inlet there is a finite current density $j_0 = \sigma_0 u_0 B_0$ which affects the stream. However, in practical cases the quantities σ_0 and B_0 are small in comparison with their final values, and the parameter S calculated on the basis of the inlet values is also small.

The second group of boundary conditions is determined on the front by formulas (1.8)-(1.10). For the dimensionless quantities we have

$$\rho_w = 1; \quad p_w = u_w^2; \quad w = \frac{\gamma + 1}{2} u_w.$$

The Euler number Eu in the present case is constant and equal to $(\gamma - 1)/2$.

2. Solution of the System of Equations. Numerical Results. The simultaneous solution of Eqs. (1.11)-(1.15) with the boundary conditions (1.16), (1.17) was carried out as follows. The xt phase plane was subdivided by a rectangular grid into cells. The cell dimension Δt was kept constant. The dimension Δx_i was defined at each time step as $w_i \Delta t$. Thus, on the wave front we always had the points (i, i) (Fig. 2). The calculation was carried out from bottom to top and from left to right in the xt plane. The subsystem (1.11)-(1.14) was solved numerically along the characteristic intervals by Euler's method, which enabled us to find the values of u, p, ρ, B at the point (i, k) from the corresponding values of the quantities at the three adjacent nodal points (i − 1, k), (i − 1, k − 1), (i, k − 1). The parameters at the points a, b, c, from which emanate the characteristics II, III, IV intersecting at the point (i, k), were found by linear interpolation between the corresponding quantities at the nodal points. The conductivity σ at the point (i, k) was determined from Eq. (1.7), and the stream density was determined from Eq. (1.15). The preliminary value of j was estimated as

$$j_{ik} = \sigma_{ik}\left(u_{ik}B_{ik} + \frac{\Phi_{ik} - \Phi_{i,k-1}}{\Delta t}\right),$$

where the magnetic flux Φ_{ik} was found by numerical integration of B with respect to x. Then, after passing to the (k + 1)-th layer, we found the value of j more precisely by the formula

$$j_{ik} = \sigma_{ik}\left(u_{ik}B_{ik} + \frac{\Phi_{i,k+1} - \Phi_{i,k-1}}{2\Delta t}\right).$$

The algorithm for finding the parameters on the wave front is somewhat different. This is due to the fact that characteristics II and IV "do not catch up with" the front, and therefore Eqs. (1.12), (1.14) cannot be used there. Instead of them, we have acting at the front the first two relations of (1.17). The third relation of (1.17) gives us the speed of the front w and the trajectory of the wave on the xt diagram within the limits of each successive time step.

The calculations were carried out for $\gamma = S/3$, $S=0.05$, and $Re_m = 5$. Figures 3-7 show the variation of the parameters in the gas behind the wave front for $\sigma = 1$ and $\sigma = T^{3/2}p^{-1/2}$ (the solid and dashed curves, respectively) for various values of t [curve 1) t = 0.3; 2) t = 0.4; 3) t = 0.5; 4) t = 0.6]. The temperature T was defined as p/ρ. The figures show how the wave is decelerated by the magnetic field and how behind the front there is formed a reflected compression wave whose intensity increases with time. It may be assumed that as Re_m increases (while preserving finite values of S), this wave becomes a reflected shock wave whose behavior is described, for example, in [10].

A characteristic feature is the formation of a zone with a temperature maximum that increases with time, moving behind the wave front. The variation of σ as a function of T and p strengthens the intensity of the interaction of the wave with the magnetic field and leads to the formation of a zone with maximum j behind the front. Additional calculations showed that when $\sigma = T^3p^{-1/2}$, the indicated effects are even more pronounced.

As was to be expected, the intensification of the magnetic field ahead of the wave front is considerably weaker than for the same values of Re_m in stationary flow, when for closure of the electrodes on the right the field increases with length like $e^{Re_m x}$. This is attributable to the action of the vortex electric field which makes j increase more slowly with time.

The accuracy of the calculations was verified by selective substitution of the numerical results obtained into the original system (1.1)-(1.6). For a step $\Delta t = 0.025$ the average of the maximum error was $\sim 10\%$ of the terms with smallest absolute value. The computation was continued until the magnetic induction on the front reached the limiting values. The computation for one regime took 15 min of machine time on the M-220 electronic computer.

For large values of Re_m and S the algorithm can easily be improved by introducing iteration cycles.

The authors are grateful to S. M.-A. Koneev for his valuable comments on the work.

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